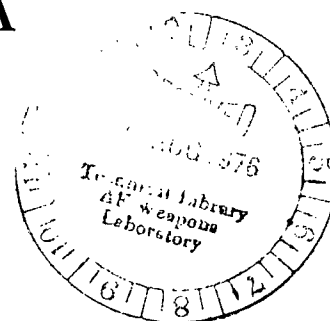# NASA TECHNICAL NOTE

NASA TN D-8286

ON ESTIMATING
GRAVITY ANOMALIES
FROM GRADIOMETER DATA

P. Argentiero and R. Garza-Robles

Goddard Space Flight Center
Greenbelt, Md. 20771

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JULY 1976

0134082

1. Report No. NASA TN D-8286	2. Government Accession No.	3. Recipient s Catalog No.
4. Title and Subtitle On Estimating Gravity Anomalies from Gradiometer Data		5. Report Date July 1976
		6. Performing Organization Code 932
7. Author(s) P. Argentiero and R. Garza-Robles		8. Performing Organization Report No. G-7684
9. Performing Organization Name and Address Goddard Space Flight Center Greenbelt, Maryland 20771		10. Work Unit No. 681-02-01-03
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		13. Type of Report and Period Covered Technical Note
		14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

The Gravsat-gradiometer mission involves flying a gradiometer on a gravity satellite (Gravsat) which is in a low, polar, and circular orbit. This document presents the results of a numerical simulation of the mission which demonstrates that, if the satellite is in a 250-km orbit, 3- and 5-degree gravity anomalies may be estimated with accuracies of 0.03 and 0.01 mm/s^2 (3 and 1 mgal), respectively. At an altitude of 350 km, the results are 0.07 and 0.025 mm/s^2 (7 and 2.5 mgal), respectively. These results assume a rotating type gradiometer with a 0.1 -etvos unit accuracy (1 etvos unit = 10^{-9} gal/cm). The results can readily be scaled to reflect another accuracy level.

17. Key Words (Selected by Author(s)) Mean-gravity anomalies, Gravity gradiometer, Least-squares estimation	18. Distribution Statement Unclassified—Unlimited Cat. 46	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 17	22. Price* $3.25

This document makes use of international metric units according to the Systeme International d'Unites (SI). In certain cases, utility requires the retention of other systems of units in addition to the SI units. The conventional units stated in parentheses following the computed SI equivalents are the basis of the measurements and calculations reported.

CONTENTS

ON ESTIMATING GRAVITY ANOMALIES
FROM GRADIOMETER DATA

P. Argentiero
R. Garza-Robles
Goddard Space Flight Center

INTRODUCTION

The Gravsat-gradiometer mission as described in Reference 1 involves flying a gradiometer on a gravity satellite (Gravsat) which is in a low, polar, and circular orbit. The low-frequency terms of the geopotential field are observable from satellite perturbations as viewed by a high satellite or satellites. If the low-satellite orbit has an altitude of 300 km, a global distribution of satellite-to-satellite tracking data can be obtained in about 10 days. From such a data set, an accurate and well-conditioned determination of the low-order terms of a spherical harmonic expansion of the geopotential field should be obtainable (Reference 2).* Because it measures second derivatives of the geopotential field, the gradiometer will be sensitive to the density variations of the Earth's outer crust, which are the source of the high-frequency components of the geopotential field. These high-frequency terms tend to be smoothed out in satellite perturbation data, and, in this sense, the gradiometer as a data type complements rather than competes with satellite-to-satellite tracking data.

The difficulty in using standard-estimation techniques to determine global geopotential fine structure is that a large number of parameters must be estimated. For instance, if features of the geopotential as small as 3 degrees are to be recovered and if the standard spherical harmonic expansion of the geopotential field is used as a parameterization, a full set of coefficients to degree and order 60 is required. This implies the estimation of over 3700 parameters. In general, simultaneous estimation of such large parameter sets is not possible. In practice, use of standard estimation techniques to recover geopotential fine structure from gradiometer data requires the adjustment of small subsets of parameters and constraining other parameters to *a priori* values. But, unless the parameterization exhibits a certain orthogonality property in the data set, the uncertainities in the unadjusted terms will badly corrupt the estimates of the adjusted terms. If spherical harmonic coefficients are used, the estimation problem can be decomposed into estimation problems of smaller dimensionality only if the gradiometer data set is globally distributed. This implies a heavy computational load. A parameterization of the geopotential field is desired in which the individual parameters exhibit a localized observability pattern in gradiometer data. With such a parameterization,

---

it would be possible to accurately estimate small subsets of parameters by processing localized blocks of gradiometer data. The mean free-air gravity anomaly-Stokes' function parameterization of geopotential fine structure (Reference 3) possesses a measure of orthogonality in local blocks of altimeter data (Reference 4). Reference 5 demonstrated the observability of gravity anomalies in gradiometer data, but the orthogonality property was assumed rather than demonstrated. Realistically, the orthogonality is not perfect and only the estimates of gravity anomalies which are a sufficient distance from unadjusted gravity anomalies will be of value. This implies that gravity anomalies should be estimated in blocks with the estimates of anomalies in a sufficiently small inner core accepted as valid and the rest should be rejected because of aliasing. To obtain an intelligent estimation strategy for determining gravity anomalies from gradiometer data, it is necessary to determine for a given data-block size the relationship between the accuracy with which a given anomaly is estimated and its distance from the nearest unadjusted anomaly.

In this document, Gravsat-gradiometer missions at altitudes of 250 and 350 km are simulated, and optimal data-block and estimation-block sizes for determining 3- and 5-degree gravity anomalies are obtained by covariance analysis. A rotating-type gradiometer as described in Reference 6 is assumed to be mounted on the Gravsat and to have an accuracy of 0.1 etvos unit. However, the results can be readily scaled to reflect some other accuracy level.

The rotating gradiometer is an instrument which senses gravity gradients in a given plane. If the instrument is onboard a satellite, that plane is assumed to be the orbital plane of the satellite. A local satellite-centered coordinate system (I_1, I_2, I_3) is defined where the I_1 unit vector is pointing northward, the I_2 unit vector is pointing eastward, and the I_3 unit vector is perpendicular to the plane spanned by I_1 and I_2 and is directed outward from the earth. If the satellite on which the rotating gradiometer is mounted is in a polar orbit, the sensing plane of the instrument always coincides with the plane spanned by the I_2 and I_3 unit vectors. This assumption simplifies the following mathematical development. The geopotential field can be represented as

$$W = U + T \tag{1}$$

where U is a reference geopotential generally defined by a low degree and order spherical harmonic expansion of a nominal field and T is the so-called anomalous potential. The anomalous potential represents the high-frequency part of the field, and the primary interest is the portion of the gradiometer output which is attributable to T. Assuming that the satellite is in a polar orbit and neglecting off-diagonal elements of the gravity-gradient tensor which are several orders of magnitude smaller than the diagonal elements

2

(Reference 5), the additive component of the rotating gradiometer signal amplitude which is attributable to the anomalous potential T is

$$
AMP = \left| \frac{\partial^2 T(r, \phi, \lambda)}{\partial r^2} - \frac{1}{r^2} \frac{\partial^2 T(r, \phi, \lambda)}{\partial \phi^2} \right| \tag{2}
$$

where r, ϕ, and λ have the usual spherical coordinate interpretations and the derivatives are evaluated at the satellite position (Reference 5).

The anomalous potential T at any point above the surface of the earth can be expressed by the discrete form of Stokes' formula as

$$
T(r, \phi, \lambda) = \sum_i \delta g_i(\phi_i', \lambda_i') \, S(r, \phi, \lambda, \phi_i', \lambda_i') \cos (\phi_i') \, \Delta\phi_i' \, \Delta\lambda_i' \tag{3}
$$

where $\delta g_i (\phi_i', \lambda_i')$ is a mean gravity anomaly over a block centered at latitude ϕ_i' and longitude λ_i' and referenced to a geoid defined by the nominal field U. The expression $\cos (\phi_i') \, \Delta\phi_i' \, \Delta\lambda_i'$ represents the area of the block on which the ith gravity anomaly is defined and $S(r, \phi, \lambda, \phi_i', \lambda_i')$ is Stokes' function given in Reference 3 by

$$
S(r, \phi, \lambda, \phi', \lambda') = t \left[\frac{2}{D} + 1 - 3 \, D - t \cos (\psi) \left(5 + 3 \ln \left[\frac{1 + t \cos (\psi) + D}{2} \right] \right) \right] \tag{4}
$$

where

$$
\psi = \cos^{-1} \left[\sin (\phi) \sin (\phi') + \cos (\phi) \cos (\phi') \cos (\lambda' - \lambda) \right]
$$

$$
t = \frac{R}{r}
$$

$$
D = (1 - 2 \, t \cos (\psi) + t^2)^{\frac{1}{2}}
$$

where R is the mean radius of the Earth. The functional relationship between the anomalous part of the gradiometer output and a globally distributed set of gravity anomalies is obtained from equations 2 and 3 as

$$
AMP = \sum_i \left| \frac{\partial^2 S(r, \phi, \lambda, \phi_i', \lambda_i')}{\partial r^2} - \frac{1}{r^2} \frac{\partial^2 S(r, \phi, \lambda, \phi_i', \lambda_i')}{\partial \phi^2} \right| \cos (\phi_i') \, \Delta\phi_i' \, \Delta\lambda_i' \, \delta g(\phi_i', \lambda_i') \tag{5}
$$

3

Equation 5 shows that, if the discrete form of Stokes' formula is valid, the gradiometer output is a linear function of a global distribution of gravity anomalies. Therefore, standard parameter-estimation techniques should apply to the problem of determining gravity anomalies from gradiometer data. To understand the application of these techniques, it is helpful to use matrix notation. Let $\tilde{y}$ be a vector of the anomalous components of gradiometer readings. Assume that the ith component of y is the anomalous gradiometer reading obtained when position of the satellite is given in spherical coordinates as (r_i, ϕ_i, λ_i). Let $\tilde{g}$ be a vector of numerical values of a global set of gravity anomalies. The jth component of $\tilde{g}$ is the numerical value of the gravity anomaly centered at latitude ϕ_j' and longitude λ_j'. The functional relationship between $\tilde{y}$ and $\tilde{g}$ is expressed as

$$\tilde{y} = A\tilde{g} \tag{6}$$

where A is a matrix the number of whose rows is the number of observations and the number of whose columns is the number of gravity anomalies. The element in the ith row and j^{th} column of A is

$$A(I, j) = \left| \frac{\partial^2 S(r, \phi_i, \lambda_i, \phi_j', \lambda_j')}{\partial r_i^2} - \frac{1}{r_i^2} \frac{\partial^2 S(r_i, \phi_i, \lambda_i, \phi_j', \lambda_j')}{\partial \phi_i^2} \right| \cos(\phi_j') \Delta\phi_j' \Delta\lambda_j' \tag{7}$$

Equation 7 provides a linear equation of condition and, in a standard minimum variance fashion $\tilde{g}$, could be estimated from observations of $\tilde{y}$. In order for equation 7 to be correct (correct, that is, assuming that the approximations inherent in the discrete form of Stokes' formula are valid), the gravity anomalies in the array $\tilde{g}$ must cover the globe. Computational considerations make it necessary to choose a region in which the number of gravity anomalies to be estimated does not exceed a few hundred. In effect, gravity anomalies outside of this region are assumed to be zero. To see precisely what happens when this assumption is made, postulate that the $\tilde{g}$ array of equation 6 is to be defined over the globe, and write

$$g = \left[\frac{\tilde{g}_1}{\tilde{g}_2} \right]$$

where

$\tilde{g}_1$ = gravity anomalies to be adjusted in a standard minimum-variance filter and

$\tilde{g}_2$ = gravity anomalies assumed to be zero and therefore left unadjusted by the minimum-variance filter

Then equation 6 can be written

$$\tilde{y} = A_1\tilde{g}_1 + A_2\tilde{g}_2 \tag{8}$$

4

where A_1 and A_2 are the variational matricies of $\widetilde{y}$ with respect to g_1 and g_2, respectively. The gradiometer output provides direct observations y of $\widetilde{y}$ with statistics

$$y = \widetilde{y} + \nu, \; E\,(\nu) = \overline{0}, \; E\,(\nu\,\nu^T) = Q \tag{9}$$

Estimates of mean free-air gravity anomalies obtained from satellite tracking and gravimetry measurements are available (Reference 7). Unless this information was correctly factored into the gravity anomaly estimates obtained from gradiometer data, the resultant estimates would not be optimal. Consequently, the existence of an *a priori* estimate g_1' or $\widetilde{g}_1$ is assumed with statistics

$$g_1' = \widetilde{g}_1 + \alpha_1, \; E\,(\alpha_1) = \overline{0}, \; E\,(\alpha_1\,\alpha_1^T) = P_1 \tag{10}$$

For computational reasons, the gravity anomalies $\widetilde{g}_2$ are assumed to be zero, but the actual values of gravity anomalies in the region of the sphere which is ignored have a certain distribution about zero. Assume that

$$E\,(\widetilde{g}_2) = \overline{0}, \; E\,(\widetilde{g}_2\,\widetilde{g}_2^T) = P_2 \tag{11}$$

When the values of $\widetilde{g}_2$ are assumed to be zero, the minimum variance estimate of $\widetilde{g}_1$ becomes

$$\widehat{g}_1 = (A_1^T Q^{-1} A_1 + P_1^{-1})^{-1} (A_1^T Q^{-1} y + P_1^{-1} g_1') \tag{12}$$

Define the covariance matrix of the estimator given by equation 12 as

$$P = E\,([\widehat{g}_1 - \widetilde{g}_1]\,[\widehat{g}_1 - \widetilde{g}_1]^T) \tag{13}$$

From equations 9, 10, 11, and 12,

$$\widehat{g}_1 - \widetilde{g}_1 = (A_1^T Q^{-1} A_1 + P_1^{-1})^{-1} (- A_1^T Q^{-1} A_2 \widetilde{g}_2 + A_1^T Q^{-1} \nu + P_1^{-1} \alpha_1) \tag{14}$$

Equation 14 yields

$$P = (A_1^T Q^{-1} A_1 + P_1^{-1})^{-1} + (A_1^T Q^{-1} A_1 + P_1^{-1})^{-1} A_1^T Q^{-1} A_2 P_2 A_2^T Q^{-1} A_1 (A_1^T Q^{-1} A_1 + P_1^{-1})^{-1} \tag{15}$$

Assume that the data noise is uncorrelated and that each data point has the same variance. Then

$$Q = I\sigma_0^2 \tag{16}$$

5

where I is the identity matrix and σ_0^2 is the common variance of the data. On the assumption that the values of the unadjusted gravity anomalies are independently distributed, the covariance matrix P_2 of $\widetilde{g}_2$ can be written as

$$P_2 = \begin{bmatrix} \sigma_1^2 & & & & \\ & \sigma_2^2 & & 0 & \\ & & \cdot & & \\ & 0 & & \cdot & \\ & & & & \cdot & \\ & & & & & \sigma_{n_2}^2 \end{bmatrix} \tag{17}$$

where n_2 is the number of unadjusted gravity anomalies and σ_i^2 is the second moment about zero of the ith unadjusted gravity anomaly. Also, define a matrix K as

$$K = (A_1^T Q^{-1} A_1 + P_1^{-1})^{-1} A_1^T Q^{-1} A_2 \tag{18}$$

If n_1 is the number of adjusted parameters, then K is of dimension n_1 by n_2. With these assumptions, equation 15 yields the following expression for the variance of the ith adjusted gravity anomaly

$$P(I, I) = \sum_{j=0}^{n_2} (\beta_{i,j}\, \sigma_j)^2 \tag{19}$$

where $\beta_{i,0}^2$ is the ith diagonal element of the matrix $(A_1^T A_1)^{-1}$ (assuming that diagonal elements of the matrix P_1^{-1} are relatively small) and

$$\beta_{i,j} = K(I,j), j \geqslant 1 \tag{20}$$

Define the error sensitivity matrix as

$$S = \left\{ \beta_{i,j} \right\}, i = 1, 2, \ldots, n_1, J = 0, 1, 2, \ldots, n_2 \tag{21}$$

And finally define the alias matrix as

$$L = S\,\bar{\sigma} \tag{22}$$

6

where

$$\bar{\sigma} = \begin{bmatrix} \sigma_0 & & & & \\ & \sigma_1 & & & 0 \\ & & \sigma_2 & & \\ & 0 & & \ddots & \\ & & & & \sigma_{n_2} \end{bmatrix}$$

(23)

The alias matrix reveals much of the probability structure of the estimation procedure. From equations 19, 21, 22, and 23 it can be seen that the standard deviation of the ith adjusted gravity anomaly is the root sum square (RSS) of the terms in the ith row of the alias matrix. The elements in the first column of the alias matrix represent the RSS contribution to the standard deviation of each estimated parameter due to the data noise. The elements in the jth column, $j \geq 2$ represent the RSS contribution to the standard deviation of each estimated parameter attributable to the j - 1st unadjusted parameter. These terms are called *the aliasing contributions* to the uncertainty in the adjusted parameters attributable to the uncertainty of the j - 1st unadjusted parameter. Note that the aliasing contributions attributable to the jth unadjusted parameter are proportional to the standard deviation of the jth parameter.

DEFINITION

In a given estimation process, the ith estimated parameter is said to be *orthogonal* with respect to the jth unestimated parameter if the aliasing contribution to the ith estimated parameter attributable to the uncertainty of the jth unestimated parameter is zero. It will be seen that the relation is symmetric in the sense that, if one parameter is orthogonal with respect to another and if they are interchanged within the adjusted and unadjusted modes, they are again orthogonal.

To understand the implications of the orthogonality relationship, a more revealing representation of the aliasing terms is necessary. Note that the first term on the right side of equation 15 is the covariance matrix of the estimation process under the assumption that the unadjusted parameters are perfectly known. This covariance matrix gives the uncertainty of the estimates which are attributable to the data noise only. Define the so-called "noise-only" covariance matrix as

$$P = (A_1^T Q^{-1} A_1 + P_1^{-1})^{-1}$$

(24)

Next, observe that the elements in the ith row and j^{th} column of A_1 and A_2, respectively, are the partial derivatives of the ith data point with respect to the jth adjusted parameter and the partial derivatives of the ith data point with respect to the jth unadjusted parameter. The aliasing contribution to the ith adjusted parameter attributable to the jth unadjusted parameter can be written as

$$L(I, j+1) = \sum_{k=1}^{n_1} \overline{P}(I, k) \sum_{\ell=1}^{m} \frac{\partial \widetilde{y}(\ell)}{\partial \widetilde{g}_1(k)} Q^{-1}(\ell, \ell) \frac{\partial \widetilde{y}(\ell)}{\partial \widetilde{g}_2(j)} \qquad (25)$$

where m is the number of data points. If the estimates of the adjusted parameters are relatively uncorrelated in the noise-only covariance matrix, equation 25 can be approximated by

$$L(I, j+1) = \overline{P}(I, I) \sum_{\ell=1}^{m} \frac{\partial \widetilde{y}(\ell)}{\partial \widetilde{g}_1(I)} Q^{-1}(\ell, \ell) \frac{\partial \widetilde{y}(\ell)}{\partial \widetilde{g}_2(j)} \qquad (26)$$

Equation 26 shows that the aliasing contribution is symmetric in its arguments, and this implies the symmetry of the orthogonality relationship.

A sufficient condition for the left side of equation 25 to approximate zero is for the observability patterns of $\widetilde{g}_1$ (I) and $\widetilde{g}_2$ (j) in the data to be virtually nonoverlapping. Figures 1 and 2 demonstrate that the perturbation of a gradiometer observation caused by an 0.01-mm/s^2 (1-mgal)* perturbation of a gravity anomaly rapidly attenuates with increasing spherical radius. Therefore, if the grids on which $\widetilde{g}_1$ (I) and $\widetilde{g}_2$ (j) are defined are sufficiently separated, the orthogonality relationship will be effectively satisfied, and the estimate of $\widetilde{g}_1$ (I) would experience no aliasing from the uncertainty of g_2 (j). Conversely, if the grid on which g_1 (I) was defined was close to grids whose gravity anomalies were adjusted, serious aliasing of the resultant estimate would probably occur.

Therefore, if the gravity anomalies are estimated in a block, the outer layers of the block contain gravity anomalies whose estimates will be badly aliased by the adjacent unadjusted parameters. It will be necessary to discard these estimates. However, the gravity anomalies in a sufficiently small inner core of the block may be adequately separated from the unadjusted parameters to be effectively orthogonal with respect to them. The estimates of these terms will presumably be of sufficient accuracy to be accepted. In effect, for every block of gravity anomalies to be estimated, it will be necessary to construct a "buffer zone" several layers deep of gravity anomalies which surround the block. The new and

---

*Throughout the text of this document, the measurement unit of milligalileo (mgal) has been converted to a Standard International Unit of millimeters/sec^2 (mm/s^2). For convenience, the illustrations have not been converted. The simple conversion is 1 mgal = 0.01 mm/s^2.
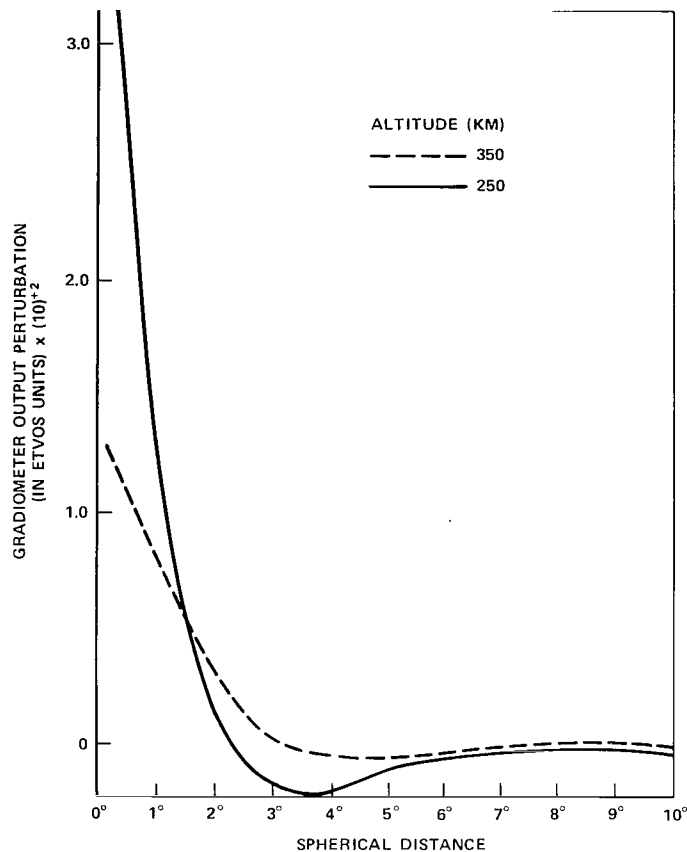
8

Figure 1. Gradiometer output perturbation attributable to 0.01-mm/s^2 (1-mgal) perturbation of 3-degree gravity anomaly.

larger block of gravity anomalies must be simultaneously estimated and because of the estimates of gravity anomalies in the buffer zone must then be rejected aliasing. To design an intelligent data-processing procedure for the Gravsat-gradiometer mission, the relationship between the depth of the buffer zone and the accuracy of the estimation procedure must be determined. This relationship will vary with grid size, data-block size, and satellite altitude. The most efficient way to study the relationship is to use covariance-analysis techniques to generate alias matricies for several situations and to attempt generalizations from the results. This method is explained in the next section.

RESULTS

Assuming that Gravsat altitude is approximately 300 km and that a gradiometer reading is obtained every 15 seconds, a dense, globally-distributed set of approximately 56000 gradiometer measurements is obtained within 10 days. In the low latitudes, the subearth points of the satellite at the measurement times form a grid on the Earth of 1 degree latitude by 1-1/2 degrees longitude. All simulations rely on this assumption concerning the
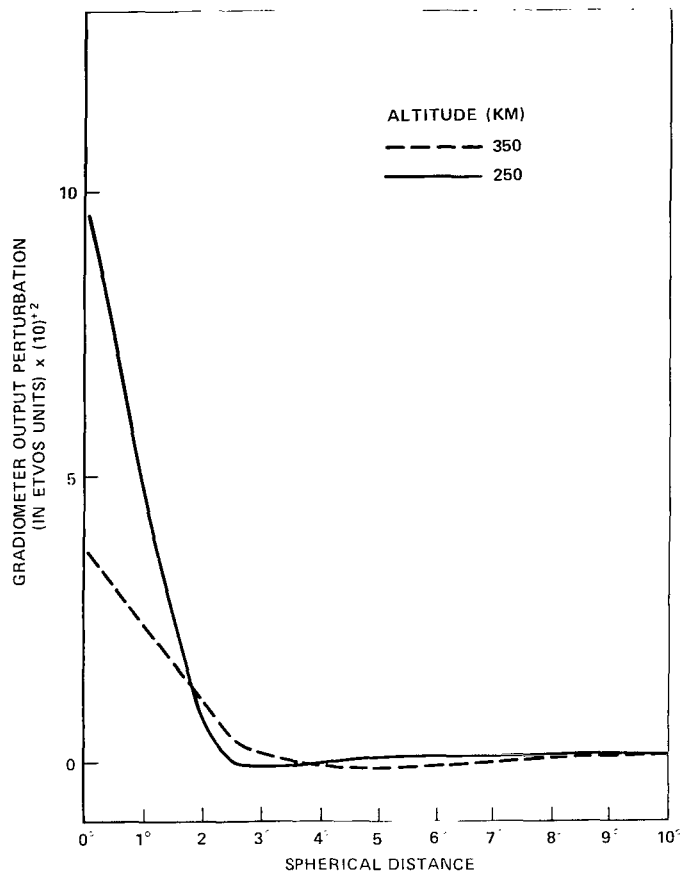
9

Figure 2. Gradiometer output perturbation attributable to
0.01-mm/s^2 (1-mgal) perturbation of 5-degree gravity anomaly.

data distribution. The dynamical aspect of the problem is therefore eliminated, and the
aliasing effect of orbit-determination errors are not included in the results. This shortcoming
is not serious. A Jet Propulsion Laboratories (JPL) study (Reference 8) indicates that,
for an instrument with an accuracy of 0.1 etvos unit, orbital accuracies of 20 to 25 meters
are adequate. Satellite-to-satellite tracking of Gravsat from a high relay satellite should
yield orbit-determination accuracies well within this range (References 2 and 9).

As mentioned in the introduction, the reason for using gravity anomalies to describe geo-
potential fine structure is the possibility of successfully estimating local blocks of gravity
anomalies in local blocks of gradiometer data. To investigate this possibility, attention
is focused on the estimation accuracy of a single gravity anomaly as a function of the
number of adjacent gravity anomalies which are simultaneously adjusted and the quantity

10

of data which is used in the estimation. The estimation block is defined as a block of gravity anomalies centered at the gravity anomaly in question and square in the sense that the block subtends the same number of degrees longitude as latitude. All the gravity anomalies in the estimation block are assumed to be simultaneously adjusted in a given estimation procedure. As defined, estimation blocks can be incremented only by integral numbers of twice the gravity-anomaly block size. For 3-degree gravity anomalies, for instance, possible estimation block sizes are 3, 9, 15, 21 degrees, etc.

The data block is defined as a gradiometer data set which was obtained when the sub-Earth points of the satellite were within a square region of the Earth's surface centered at the gravity anomaly in question. For a given estimation-block size, the accuracy of a gravity anomaly estimate is not necessarily a monotone function of the data-block size. At first, this may be puzzling because better estimates are generally expected from increased data. An explanation can be obtained by noting that for a given estimation block the covariance matrix of the set of estimated gravity anomalies is given by equation 15 as the sum of a matrix which is dependent only on data uncertainty and a matrix which represents the aliasing effects from the unadjusted parameters. With increasing data blocks, the elements of the first matrix must decrease, but the elements of the matrix which conveys the aliasing effects will generally increase. This effect can be shown graphically by means of so-called aliasing maps. To obtain the aliasing maps of figures 3 and 4, in a covariance mode a situation was simulated in which the geopotential fine structure was described by means of 3- by 3-degree gravity anomalies and Gravsat-gradiometer data with an accuracy of 0.1 etvos unit was assumed. In figure 3, the RSS contribution was mapped in mgals to the uncertainty in the estimate of the gravity anomaly defined on the blank grid element in the middle of the graph when the adjacent gravity anomalies were assumed to be in an unadjusted mode and to have an *a priori* uncertainty of 0.50 mm/s² (50 mgal). The data block represented by the shaded area of the graph was 10 degrees on a side. The satellite altitude was 250 km. Note that the aliasing contributions decrease with the distance between an unadjusted parameter and the estimated parameter, thus demonstrating the inherent orthogonality properties of gravity anomalies in gradiometer data. In figure 4, the data-block size has been increased to 20 degrees, and the aliasing effect of adjacent unadjusted gravity anomalies is greater.

The choice of an intelligent data-reduction strategy is obviously dependent on a knowledge of the relationship between estimation accuracy and the choice of data-block size and estimation-block size. A computer program was written which computes the right side of equation 15 for any given data-reduction technique. The program was used to investigate data-reduction strategies. First, an altitude for Gravsat was chosen and a gradiometer data block centered on a chosen gravity anomaly was assumed. The data accuracy was assumed to be 0.1 etvos unit. A normal matrix for 400 gravity anomalies was formed. The chosen gravity anomaly and gravity anomalies in successive layers surrounding the chosen gravity anomaly were assumed to be in an adjusted mode, and the rest were placed in an unadjusted mode with uncertainties about zero of 0.50 mm/s² (50 mgal). The right side of equation 15 was computed, and the resultant standard deviation in the estimate of

11

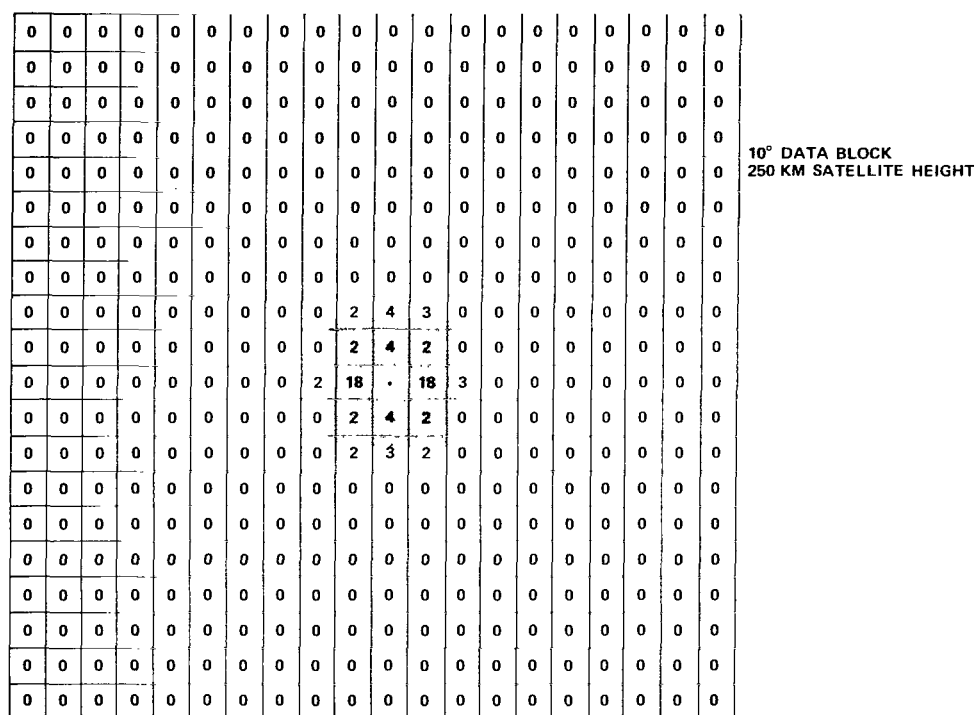| 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	2	4	3	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	2	4	2	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	2	18	·	18	3	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	2	4	2	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	2	3	2	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

10° DATA BLOCK
250 KM SATELLITE HEIGHT

Figure 3. Alias map for 3-degree gravity anomalies.

the chosen gravity anomaly was recorded. The estimation-block size was incremented by removing another layer of gravity anomalies from the unadjusted to the adjusted mode and repeating the process. Figures 5 and 6 show the relationship between estimates of 3- and 5-degree gravity anomalies and the choices of data-block size and estimation-block size when Gravsat is at an altitude of 250 km. Figures 7 and 8 show the same information when Gravsat is at 350 km. The graphs show that an intelligent compromise between computational load and accuracy is achieved when a data-block size of 15 degrees is used and the buffer zone of rejected gravity-anomaly estimates is at least three layers deep. The graphs show that when this strategy is followed and when the satellite is in a 250-km orbit, 3- and 5-degree gravity anomalies can be estimated with uncertainties of 0.03 and 0.01 mm/s^2 (3 and 1 mgal), respectively. When the satellite is in a 350-km orbit, 3- and 5-degree gravity anomalies can be estimated with uncertainties of approximately 0.07 and 0.025 mm/s^2 (7 and 2.5 mgal), respectively.

These results are dependent on the chosen value for the accuracy of the rotating gradiometer. Because this choice is somewhat arbitrary, it is useful to parametrically represent gravity-anomaly-estimation accuracy as a function of gradiometer accuracy. Equation 19 implies that, for a given choice of estimation-block size and data-block size, the standard deviation of a gravity-anomaly estimate is the root sum square of a term which represents the aliasing effects of unadjusted gravity anomalies and a term which is proportional to

12

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	2	2	2	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	2	4	5	4	2	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	2	4	3	9	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	4	19	.	19	4	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	2	4	2	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	2	4	8	4	2	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	2	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

20° DATA BLOCK
250 KM SATELLITE HEIGHT

Figure 4. Alias map for 3-degree gravity anomalies.

the standard deviation of the rotating gradiometer observation. Assuming a 15-degree data-block size and assuming that a given gravity anomaly is simultaneously estimated with all gravity anomalies in the three adjacent layers, the estimation accuracies for 3- and 5-degree gravity anomalies at satellite altitudes of 250 and 350 km are given as a function of data accuracies as

$$\sigma(3°, 250\text{km}) = (625\ \sigma^2 + 2.25)^{1/2}$$

$$\sigma(5°, 250\text{km}) = (81\ \sigma^2 + 0.5(10)^{-2})^{1/2}$$

$$\sigma(3°, 350\text{km}) = (4200\ \sigma^2 + 3.24)^{1/2}$$

$$\sigma(5°, 350\text{km}) = (625\ \sigma^2 + 0.01)^{1/2}$$

(27)

Standard deviations on the left side of equation 27 are in mgal and the data standard deviation σ on the right side of equation 27 is in etvos units.
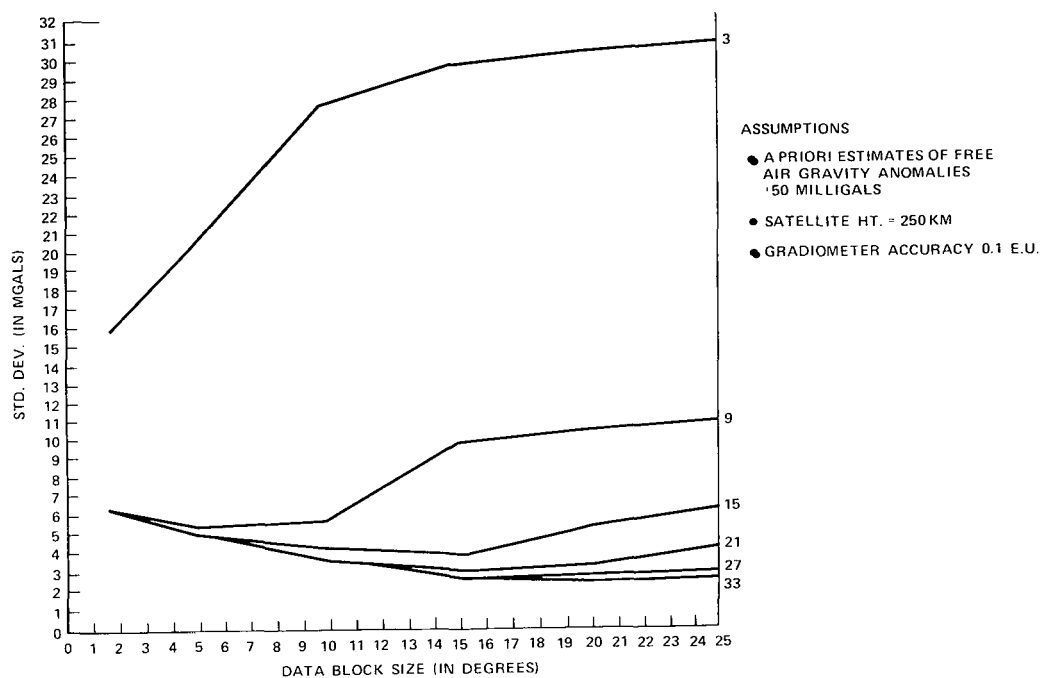
13

Figure 5. Accuracy of 3- by 3-degree mean free-air gravity anomaly estimate versus data-block size for various estimation-block sizes.
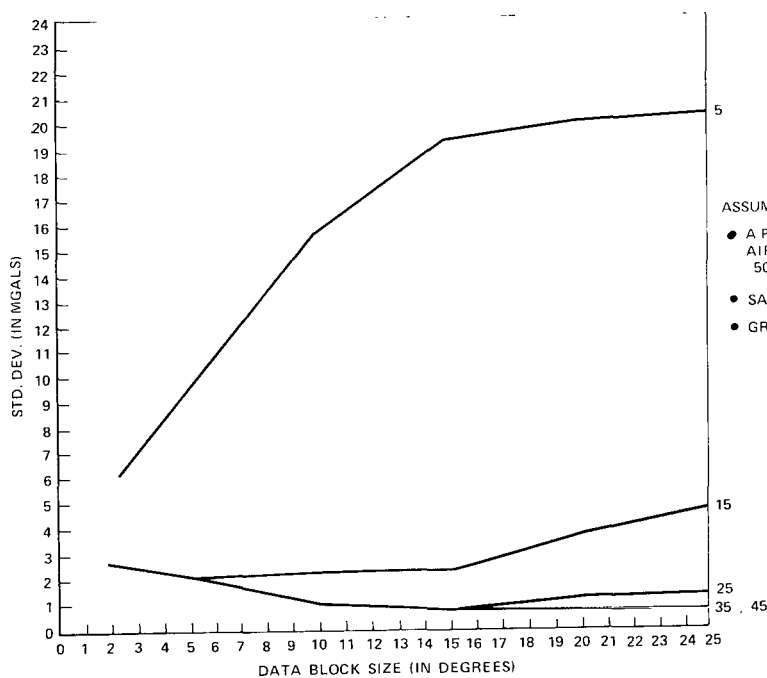

Figure 6. Accuracy of 5- by 5-degree mean free-air gravity anomaly estimated versus data-block size for various estimation-block sizes.
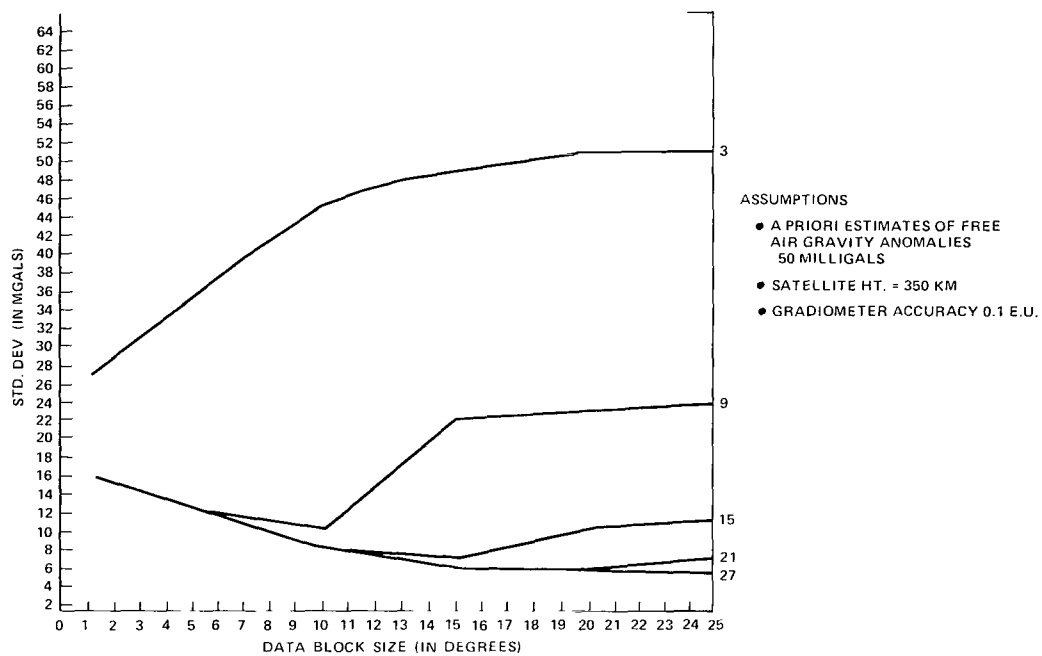
14

Figure 7. Accuracy of 3- by 3-degree mean free-air gravity anomaly estimate versus data-block size for various estimation-block sizes.
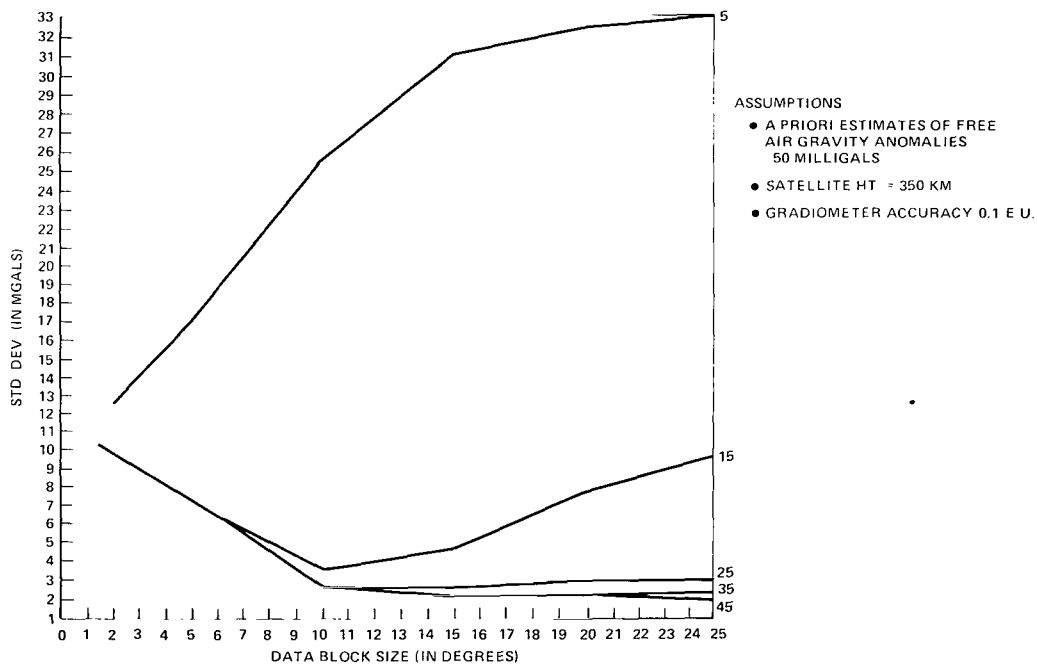

Figure 8. Accuracy of 5- by 5-degree mean free-air gravity anomaly estimate versus data-block size for various estimation-block sizes.

15

CONCLUDING REMARKS

If a rotating gradiometer were flown on a satellite in a polar, circular orbit with an altitude of 250 to 350 km, a dense and globally distributed set of gradiometer observations would be obtained in about 10 days. From this data set, it should be possible to determine a global set of gravity anomalies. However, to keep the computational load within reasonable bounds, it will be necessary to estimate local blocks of gravity anomalies in local blocks of gradiometer data. The results of this study indicate that good gravity-anomaly estimates can be obtained from gradiometer data blocks as small as 15 by 15 degrees if each gravity anomaly whose estimate is to be accepted is separated from unadjusted gravity anomalies by three layers of adjusted gravity anomalies. If the satellite is flown at an altitude of 250 km and if the instrument has an accuracy of 0.1 etvos unit, this estimation strategy yields estimates of 3- and 5-degree gravity anomalies accurate to within 0.03 and 0.01 mm/s^2 (3 and 1 mgal), respectively. When the satellite is flown at an altitude of 350 km, the accuracies of 3- and 5-degree gravity-anomaly estimates are 0.07 and 0.025 mm/s^2 (7 and 2.5 mgal), respectively. Intermediate results can be expected at intermediate altitudes. Equation 27 of this document permits a scaling of these results to a data accuracy level different from 0.1 etvos unit.

16

REFERENCES

1. Earth and Ocean Physics Applications Program, Vol. II, Rationale and Program Plans, NASA, 1972.

2. Koch, D., and P. Argentiero, *Simulation of the Gravsat/Geopause Mission,* NASA TM X-70776, 1974.

3. Heiskanen, W. A., and H. Moritz, *Physical Geodesy,* W. H. Freeman and Co., 1967.

4. Argentiero, P., W. Kahn, and R. Garza-Robles, *Strategies for Estimating the Marine Geoid from Altimeter Data,* NASA TM X-70637, 1974.

5. Reed, George B., *Application of Kinematical Geodesy for Determining the Short Wave Length Components of the Gravity Field by Satellite Gradiometry,* The Ohio State University Research Foundation, Report No. 201, March 1973.

6. Forward, R. L., *Development of a Rotating Gravity Gradiometer for Earth Orbit Applications,* Preprint and paper presented at the Advanced Applications Flight Experiment (AAFE) Program Review, NASA Research Center, Hampton, Va., October 5-6, 1971.

7. Marsh, J., and S. Vincent, *A Global Detailed Geoid,* NASA TM X-70492, 1973.

8. Gardner, J. A., et al., *Earth Physics Satellite Gravity Gradiometer Study,* Document No. 760-70, Jet Propulsion Laboratory, CIT, Pasadena, California, 1970.

9. Argentiero, P., and R. Garza-Robles, *GEOS-C Orbit Determination with Satellite-to-Satellite Tracking,* NASA TM X-70659, 1974.

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."
—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons. Also includes conference proceedings with either limited or unlimited distribution.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include final reports of major projects, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION OFFICE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546